Discrete Mathematics Project

Summer 2016

7th - 9th Grade

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Executive Summary:

This unit on Discrete Mathematics will build skills on pattern recognition, and critical thinking while using manipulatives and other group activities. The students will take an in depth look at the specific rules involved in the use of permutations and combinations. This will be accomplished by exploring real world applications in small groups. Students will work with a number of problems involving patterns and growth. In doing so,, they will develop strategies to come up with recursive and explicit algorithms to calculate the next iteration and eventually the nth iteration.

Our lessons are designed in such a way that you can pick one lesson that fits in with your curriculum. These lessons also work well when you can not avoid a substitute teacher in your room. The lessons are self guided and will allow for classroom discussion at any time. The lessons are designed to have students investigate Critical Thinking, Combinations, Permutations, and Area using Geoboards.

The following units and activities are not meant to be taught consecutively; they may work as activities before or after your units they follow in the curriculum you teach from, or they may be used as fun activities at anytime. All of the following lessons and activities have the Minnesota State Standards that they meet at the beginning of the lesson. All activities are meant to be in small groups or partners.

Table of Contents:

Counting and Area Pre Test

1. Ian has 3 pairs of pants, 5 t-shirts, and 2 baseball hats. How many different outfits can he make?

2. The school has 8 people trying out for the basketball team. Only 5 will be chosen.

a. The coach wants each player to have a turn taking free throws. How many possible orders are there for the players to take their shots? Estimate your answer.

b. How many possible teams can be made?

c. If the coach wants to choose 5 players from the 8 and give them each a unique position, how many combinations of teams can be made?

3. How many four digit numbers can you make out of the digits 1, 2, 3, & 4?

4. How many different ways can I lay 10 paving slabs, each 2 foot by 1 foot, to make a path 2 foot wide and 10 foot long from my back door into my garden, without cutting any of the paving slabs?

5. Marcus used 80 feet of fence to build a square pen for his dog. How many square feet is the area?

6. Move 2 vertices in the graph below to make a different shape.

Area Dice Game/Geoboard Area

7.3.2.1 Describe the properties of similarity, compare geometric figures for similarity, and determine scale factors.
7.3.2.2 Apply scale factors, length ratios and area ratios to determine side lengths and areas of similar geometric figures.

Materials
- Geoboard for each student
- dice (2 for each group of 2-3 students)
- full sheets of graph paper (1 inch or ½ inch)
- geoboard paper

Launch: Hand out graph paper to each group of 2-3 students and 2 dice to each group. Rules for the game each player rolls the two dice and takes each dice as one of their side lengths to draw a rectangle on their graph paper with each square being one unit. The student then colors in that square in a specific color (every student has their own color) and write the area within the square. Then the next student takes their turn, make sure you have the students add up their areas as they go so at the end the student with the largest area colored in wins.

Hand out the Geoboards and ask the students to make rectangle on their boards then have them count and record the perimeter pegs, interior pegs, and the area of the shape. Are we counting spaces or pegs?

Explore: Have the students play the area dice game through at least once and talk about if there are any strategies that they are using. Then let groups take on others from different groups or as a group play a different group. Let the students play until they come up with goals or ways in which they believe they can win.

Have the students make multiple different shapes and count up the perimeter pegs, interior pegs, and area of each of the new shapes they made. Have them draw the shapes on the geoboard paper that they want to keep. Then make a class table of every groups/student contributing one shape that they found. Is there a pattern or a way of finding the different pieces perimeter pegs, interior pegs, and area if you know 2 of them how can you find the 3rd one?

Share: SW share out their philosophies of how they believe they can always win if they come up with a process on the most productive way to play.

SW share out the different patterns that they found and if they found any equations. Along with the process in which they used to get to those conclusions.

Summarize: TW go over the different ways students were playing the game and as a class go over one or more ways that students had to win the game. TW talk about the patterns that were found in between the area, exterior pegs and interior pegs.

Dominoes, Dominoes, Dominoes but no pizza

9.4.3.1 Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.
9.4.3.2 Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.

Materials
-at least one set of dominoes
-a page with at least 28 dominos on it for the students to draw out their 28 dominos.

Launch: TW explain what is a Dominoe: A rectangle that is formed by two congruent shapes. With each of the squares containing an orderly pattern of dots that represent numbers 0-6. Then pose the question: Can a dominoe be made under these constraints?
Answer: students can give any example of a domino to state yes you can make domino's under those constraints.
Q: How many different dominoes can be made under these constraints? (A: 28 dominoes)

Explore: SW work through this counting problem either by drawing out the possibilities or using probability of choices. Students will work through this problem in groups of 4.

Share: SW share out their answers of how many dominoes can be made under these constraints. Some students may draw out all of the different dominoes. Those that look at and try to find out how many ways you can make a domino without drawing them all out. They may come up with 7 choices and 7 choices, 7x7=49 dominoes. Does this account for double counting? Can you have half of a domino? Are there different types of dominoes? Should be count them separately or together can we count them as one group.

Summarize: TW walk through the different process that students have made to find the answer of 28 dominoes. If they have not have the students see if they can write out all 28 dominoes. An extension of this activity Q: If you have 4 people that are going to play a game of dominoes and you need to divide the full set of dominoes (28 dominoes) equally among the four players. How many different ways can the 28 dominoes be made? Answer: this is a counting problem
28!/[(7!)(7!)(7!)(7!)]=4.7x10^14 ways
C(28,7) x C(21,7) x C(14,7) x C(7,7)= 4.7x10^14 ways

Constellations

Material

-pictures of constellations drawn in vertex edge graphs

Launch: present the class with the constellation of little dipper and ask the students to move vertices around to make a different shape.

Explore: SW work through moving the vertices around to create the different shapes in groups of 2-4 students. Students can also work with different constellations as well.

Share: SW draw their different shapes and compile them as a class on the board. Talk about how many the different shapes were made. Have we found all of the shapes? How do we know if there are more shapes or not?

Summarize: Talk about what changing the movement of one vertex does and how the number of edges attached to that vertex effects that movement. Can you trace your new shape without picking up your pencil?

Extension: trace a small section of a highway map. Make cities, towns, and crossroads. How many road can you erase and still have all the towns connected. Or you can ask them what roads can you add and still be able to drive every road and not double back on any of the roads.

Pentominoes Counting and Comparing Area

9.4.3.1 Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.
9.4.3.2 Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.
7.3.2.1 Describe the properties of similarity, compare geometric figures for similarity, and determine scale factors.
7.3.2.2 Apply scale factors, length ratios and area ratios to determine side lengths and areas of similar geometric figures.

Materials
-plastic or paper pentominoes sets for each group of students (2-4 students in a group)
Pentominoes cut outs

Launch: TW hand out pentanomial the sets to the students. Explain that the goal of the activity is to put all of their pentominoes into a rectangle using all 12 of the pentominoes.

Explore: SW will explore the different ways they can make a rectangle. Are all of the rectangles possible? (1 by 60, 3 by 20) Why are some not possible? Can one set of dimensions have more than one solution? Is a square a possible outcome? How do their areas compare?

An extension is can you know make a rectangle with a window in the middle of 4 squares. Or what if you took one of the pieces out what can you make now.

Share: SW show their different outcomes and talk about their thoughts on the different number of solutions. Have we found all of the solutions? Can you see any lines of symmetry in your rectangles? Can you make one with symmetry?

Summarize: TW talk about the different solutions that were found. That there are different solutions and discuss how the different dimensions have more or less number of solutions. If you wanted your students to be able to work with these at home have them each cut out their own set. Or before you start this activity you can also have your students try to find all 12 of the pentominoes.

So Many Squares

9.4.3.1 Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.
9.4.3.2 Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.

Materials
- dot paper
- optional geoboards
- optional checker/chess board

Launch: TW hand out dot paper and ask the students to connect a 3 dot by 3 dot square.
Q: How many squares can you count in the 3 by 3 square.

Explore: SW draw the squares and count how many squares they can make. Q: How about on a geoboard which is a 5 by 5 square? Q: What about on a checkerboard? Q: Can you count the diagonal squares? If so then how many squares are there?

Share: Groups will share their number of squares that they found and give their process. Is there a way to find any number of length squares? Is there a relationship or pattern to this process?

Summarize: What are the different levels of counting the squares? How can we be sure that we counted all of the squares. Go over the process in which we found the squares. The relationship between different sized squares.

Post-test counting and area

1. How many different outfits can you make from 4 hats, 3 jackets, and 2 pairs of pants?

2. John, Paul, and George all attend the same high school. There are 4 unique music classes available for them to take.

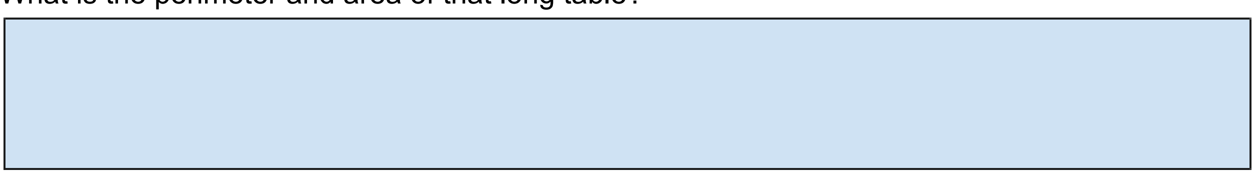
a. If the three students all end up in different classes, how many possible combinations are there?

b. If John and Paul end up in a class together, what are the chances that George will be in the same class?

c. John and Paul are in the same class, but George is in a different one. How many possible combinations of classes are there?

3. How many new combinations can be made by rearranging the letters of the word MUSIC?

4. Five Square tables with sides of 30 inches are placed end to end to form a long table. What is the perimeter and area of that long table?



5. Draw a vertex edge graph that has 7 vertices and 7 edges then move 2 vertex to make it into a different shap.

Unit: Permutations and Combinations (12 days of instruction)

MN State Standard:
9.4.3.1 Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.

MCA III (tested in 11th grade) question:

> A group of health care providers consists of 4 doctors, 3 dentists, and 5 nurses. How many combinations of 2 health care providers of different types are possible?
>
> Enter your answer in the box.

DAY 1-2 Counting Principle (Pretest and Starburst)

*Students will complete a 15 question pre test regarding Counting Principle, Permutations (Distinguishable items) and combinations.

*As students finish, have this word problem on the board:

A movie theater sells 3 sizes of popcorn (small, medium, and large) with 3 choices of toppings (no butter, butter, extra butter). How many possible ways can a bag of popcorn be purchased?
(courtesy of http://www.regentsprep.org/regents/math/algebra/apr1/praccnt.htm)

LAUNCH: Pick four volunteers to come to the front of the room. Three of the volunteers will be moving about and one will be taking notes on the board. Ask students how many different ways can the three student volunteers be arranged?

After the students are done moving about and the notes have been taken, question students about what they observe.

EXPLORE: Hand out a baggie of Starburst's to each student in their group. Then hand out worksheet that corresponds with arranging three and four different Starbursts. Students will work in their groups.

Check for understanding.

SUMMARIZE: Have a group member write how many starburst arrangements they came up with on the board. Then have a few groups show their pictures under the projector. Ask students if there is a pattern to figuring out five different colors. Talk about brute force and then describe the Fundamental Counting Principle.

ASSESSMENT: Correct pre test and starburst arrangement worksheet to see if the class can move forward with permutations.

Starburst Counting Activity

Directions:

1) Do not eat your Starbursts until you are told by teacher.
2) Find as many different arrangements using only red, yellow, and pink Starbursts.
3) Show the different arrangements you and your group come up with.

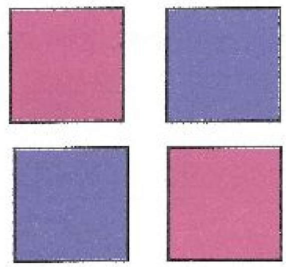
Directions:

1) Find as many different arrangements using all 4 of your Starbursts.
2) Show the different arrangements you and your group come up with.

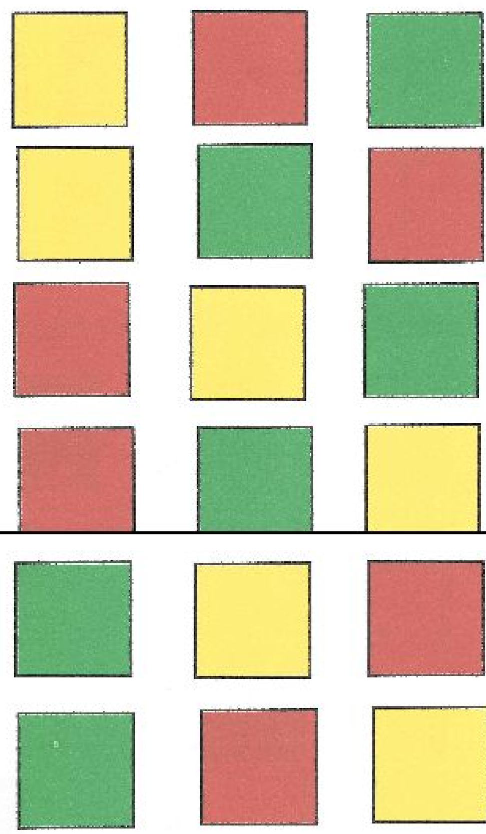
Taking a step further: How many arrangements do you think you can make using 5 different colored Starbursts?

Activity 10a -Arranging Starbursts - Permutation:

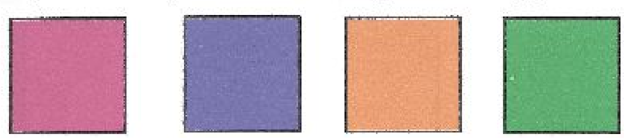
1. How many ways can you arrange 2 different colored Starbursts?



? How many ways can you arrange 3 different colored ;tarbursts?



3. How many ways can you arrange 4 different colored Starbursts?



4. Can you find a mathematical shortcut?

DAY 2 (Counting Principle continued)

LAUNCH: Have a picture of the a MN license plate on the Smart board. Ask students about license plates and what they know about them. Talk to them about the different ways license plates can be arranged. Question students about their conjecture on how many license plates arrangements they think there are.

EXPLORE: On SMART board slide have what Minnesota, California, New Hampshire require for a license plate. Have students work in their groups to come up with a number for each one of the states.

Then amp it up a little bit:

For MN: if the 2nd space can on be a vowel and the fourth place can only be #'s 1-5, now how many different arrangements?

For CA: if the 3rd space could only be a vowel and the numbers have to be prime.

For New Hampshire: The first, third and fifth place can only be even numbers while the 2nd, 4th and 6th places can only be odd...then how many arrangements.

SUMMARIZE: Ask students to compare and contrast yesterday's learning to todays.

ASSESSMENT: Correct student work on license plates.

DAYS 3 (Permutations Pizza Activity)

LAUNCH:

Talk about what does the Student Council do in a school? Ask the students who/what makes up the Student Council body for the school?

*President, Vice President, Treasurer and Secretary

Ask students: How many different ways can we arrange a class of 32?

MAKE SURE TO TALK ABOUT **DISTINGUISHED** ITEMS!

Have students put their answers on the board and explain their answer.

Give 2 more examples: How many arrangements can we make from the word BANANA?

B-1! A-3! N-2! (Distinguishable)

How many arrangements can we make from the word Minnesota?

M-1! I-1! N-2! E-1! S-1! O-1! T-1! A-1!

Write out different ways to represent Permutations.

EXPLORE: Hand out Pizza Combinations worksheet and allow students to work in their groups.

Pizza Place Combinations -- Combinations and Permutations Project

SUMMARIZE: Question students about tricky problems, hard problems and make sure to clarify their mistakes and questions.

ASSESSMENT: Correct pizza worksheet to check for understanding on permutations.

DAY 4-5 (Permutations and Combinations Mall Activity)

LAUNCH: Say we have a 23 member club at 4-H. The 4-H Headquarters three representatives from each club. How many arrangements can we make?

*Have student put their answers on the board and explain.

*Are the items distinguishable? No

Give another example: Talk about multiple choice questions. How many choices do they give? Let's say we have a 15 question multiple choice test and each question consists of 5 answers. In how many ways can a student answer each question on the test?

(cite: https://www.yumpu.com/en/document/view/2125420/permutations-and-combinations)

*Talk about how are these different then permutations?

EXPLORE (2 days): Have student work in their groups to complete the Mall Task Card Activity. Tell them they must complete have of the task cards today and the rest tomorrow.

Combination and Permutation Task Cards - Uses Mall theme, Perseverance Required!

SUMMARIZE (last 6 mins): Ask students which problems they had difficulty on.

There will be confusion as to use combinations/permutations/counting principle.

MAKE CLARIFICATIONS

ASSESSMENT: (1st day) Ticket out the door: Describe using complete sentences how a person can tell the difference between a combination, permutation and the counting principle. (2nd day) Correct the task card activity.

Days 6-8: Clever Counting The Case of the Looted Locker

(Borrowed from Clever Counting, Connected Mathematics Project c.1996, some details have been changed to be more relatable to students in today's society.)

Materials:

- Math notebook/paper
- pencil

Launch:

One of the most popular stores in Euclid Mall is Willie's Radical Sounds. The store sells radios, ipods, and mp3 players at low discount prices. Radical Sounds has very little room to store equipment that is not on display. So Willie rents a storage locker at the nearby Fail-Safe Warehouse. Unfortunately, Willie went to his storage locker one day and found that a box of expensive mp3 players was missing. Willie called the police, and a detective came to investigate. The detective quickly learned that:

1. There were no signs of forced entry and no fingerprints, except Willie's.
2. The warehouse manager was on duty from 9AM to 6PM everyday. For safety reasons, she had a list of all lock combinations.
3. The night security guard was on duty alone from 9PM to 6AM. The manager suspected that the guard opened Willie's locker by taking his time alone to try all the possible combinations.
4. The night security guard said that he did not steal the mp3 players. He admitted that he was visited by a friend from about 9:30 until about 11:30PM. He and the friend played dominoes for about an hour. Then he went off on a half-hour round of the warehouse to check all locks and doors.
5. The guard's friend was brought in for questioning. She recalled that shortly after the guard left on his rounds a van raced out of the warehouse parking lot with its lights off. Because this seemed strange, she tried to notice the license plate of the van. She caught only the first three letters, MTU.

Based on this information, whom would you suspect? What questions would you like to ask Willie, the manager, the night guard, the guard's friend, or the detective? Could the night guard, his friend, or the person who left in the van, open Willie's locker without the combination?

Explore: Investigation 1: Opening Willie's Locker

The detective thought that the warehouse manager and Willie himself were really the prime suspects. But, before making an arrest, she wanted to know if the night security guard or his friend could really open the lock by trying all the possible combinations. The combination on Willie's locker consists of 3 numbers given in order, like 15 - 5 - 33 or 7 - 11 - 21. Any numbers from 0 to 39 can be used in the combination for such a lock. Is it possible for the guard or his friend to try all possible combinations?

A. Working by yourself make a list of all the facts you would need to know to decide how much time would be necessary to open the lock, if the thief used a trial and error search through the possible combinations?
B. Compare your ideas with a partner and, if possible, use your combined list to estimate how long a trial and error search might take. If you believe that it is not possible to make an estimate then explain why.

If you were able to answer the detective's question about combination locks, you must have developed some clever counting strategy. Here are some hints about a problem solving strategy that might help you check the answer you got for A and B.

1. One way to start on a counting problem is to begin listing some examples of the things you are trying to count. It also helps to consider a simple case first.

a) Suppose the dial of a lock has only two marks - for the numbers 0 and 1 - and that combinations are three numbers in a given order, like 0 - 0 - 1 or 1 - 1 - 0. List all possible combinations if the lock has only two numbers - 0 and 1 - on its dial.

b) List all possible combinations if the lock has only three marks - for 0, 1, and 2 - on its dial. You are still making **three number** combinations.

c) List all possible combinations if the lock has only four marks - 0, 1, 2, and 3 - on its dial. Again, you are making **three number** combinations.

d) Can you now see a pattern that would predict the number of three number combinations for a lock with 5 numbers on its dial?

2. In counting problems with many possible outcomes, it helps to start a list of possibilities in an organized way to show interesting patterns. Look again at the problem of counting combinations for Willie's lock, with 40 marks on the dial for the numbers 0, 1, 2…….39.

a) Extend each of the lists below to at least ten combinations. Then look for patterns that help answer the questions that follow.

0 - 0 - 0	0 - 1 - 0	0 - 2 - 0
0 - 0 - 1	0 - 1 - 1	0 - 2 - 1
0 - 0 - 2	0 - 1 - 2	0 - 2 - 2

b) How many combinations will start with 0 - 0 - ___?

How many combinations will start with 0 - 1 - ___?
How many combinations will start with 0 - 2 - ___?
How many combinations will start with 0 - ___ - ___?

Share:
Have each group list and discuss facts they believe they would need to know to decide how much time would be necessary to open the lock. Let students explain why they do or do not believe it is possible to make an estimate. Give students time to work in small groups and then come to the board to share and discuss each new discovery. This entire lesson can be spread out over 3 days, as you keep it a full class discussion with partner explore time and then coming back together as a class to discuss all findings. Have students go to the board and share their findings as often as possible. Let groups bounce ideas off of each other and help come to their answers together.

Summarize:
Have students answer the following questions on the board to sum up the lesson thus far and see if everyone is on the same track or not.

c) Can you count the total number of possible combinations now?

d) What strategy is being used to write the list in a systematic way so that all possibilities are listed?

e) Does this help you answer the detective's question? What do you still need to know?

Days 9-12: Clever Counting The Case of the Looted Locker

(Borrowed from Clever Counting, Connected Mathematics Project c.1996, some details have been changed to be more relatable to students in today's society.)

Materials:

- Math notebook/paper
- pencil

Launch:

You have been concentrating on three-number combinations for different locks. The number of combinations depends on the number of choices for the three numbers. You may have seen a pattern that lets you list and count the combinations, or you may have seen a pattern in the results that lets you predict the number of combinations without listing. The detective has also been experimenting with simple combinations and the question she now wants to know is: It seems to take about 10 minutes to try all the three-number combinations that use 0, 1, 2 or 3. Will it take ten times as long to try all the three-number combinations that use 0, 1, 2, 3……..39?

Explore:

In a table collect all the information you found about the number of combinations for simple locks with only a few numbers on the dial. In each case, consider three-number combinations.

Number of Marks on the Dial, *m*	Number of Combinations, C
2	8
3	?
etc.	

A. What rule seems to express the relationship between C and *m*?
B. Make a sketch of the graph of this rule for values of *m* between 2 and 10. Explain how the shape of the graph relates to the question that the detective asked herself.

1. Use the rule you found to work out how many combinations a burglar might have to try to open Willie's lock.
2. How would the rule change if you had to

a. Make two-number combinations, choosing from the 40 marks on the dial?
b. Make four-number combinations, choosing from the 40 marks on the dial?
c. Make four-number combinations choosing from the 20 marks on the dial?

Share:

You will stop groups at various points to have them show their work on the board and explain what they've done so far. I go off of students needing to move around, a break between questions, and any time people seem stuck. You can never stop to discuss too often, so I put all of the explore work above and it is up to you to stop when you feel appropriate.

1) Can you write a formula to figure out the number of combinations for choosing the following. Discuss with your group and then head to the board and be prepared to explain how your group came up with it.

a) Three-number combinations from 40 marks on the dial
b) Three-number combinations from N marks on the dial
c) R-number combinations from N marks on the dial.

2) Who do you suspect has the missing mp3 players? Be prepared to explain why you answer as you do.

Summarize:

What have we found when making our lists of possible combinations? Can you answer the detective's new question? How do you know? Did making a table help you more today than the previous days that we wrote out each combination?

On day 12, finish all of the main ideas in the lesson and see if anyone still has any questions and discuss as a whole group. Then hand out the pre/post test for Combinations.

Critical Thinking Pre-test

1. Draw a vertex edge graph that has 7 vertices and 7 edges that when you move 2 edges you can make into 2 squares?

2. Show a complete magic square and explain how you got to your answer.

3. Mrs. Smith has 4 children: Anne, Tim, Katy, and Luke.

- **The youngest child is NOT a boy**
- **Luke is younger than Anne**
- **Tim is older than Anne**

Order the Children from youngest to oldest.

4.

Oder of People	Votes
H>P>C	5
H>C>P	10
P>C>H	5
P>H>C	15
C>H>P	6
C>P>H	9

Harry, Peter, and Carrie are all running for class president. When the class of 50 students voted, they rank the 3 candidates their 1st choice, 2nd, and 3rd which is shown in the table above.

a. Who would win using Plurality?

b. Who would win using Elimination?

c. Who would win using Boarda?

d. Who would win using Head to Head?

e. Which of these do you think is the most fair or unfair why?

f. What other way can we show this data to find a winner?

Who is Wearing What Color?

Materials

- logic puzzle prompts and table

Launch: TW first read through the beginning prompt:
One Saturday five friends visited the zoo. Each wore a different color t-shirt and each rushed to see their favorite animal upon arriving at the zoo. Using the clues provided, can you name each child's favorite animal and the color t-shirt they wore?
Then hand out the following clues and the table for the students to fill in.

Explore: SW work through in groups reading through all of the prompts and filling in the table provided. Is there more than one solution? What information holes did you need to fill and how did you go about that?

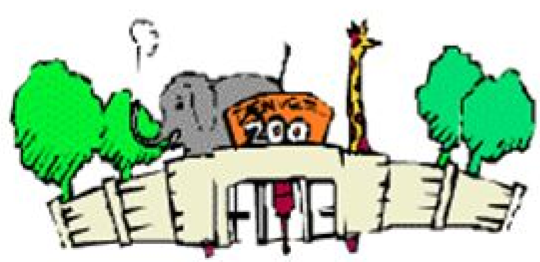
Share: SW share their outcomes of their finished table with the class and we will go through each dispute of the different parts of the table. How did you start your table out? Was there any t-shirt colors or animals that you knew from the clues had to be with a specific tool.

Summarize: TW talk about the different strategies that the class used. Confirm that there is more than one way to to successfully do this puzzle even if you did not start in the same place. Talk about the critical thinking and ask the students how they felt throughout doing this problem.

One Saturday five friends visited the zoo. Each wore a different colour t-shirt and each rushed to see their favourite animal upon arriving at the zoo. Using the clues provided, can you name each child's favourite animal and the colour t-shirt they wore?

1. One of the children wore the t-shirt that was the same colour as their favourite animal.
2. Steven, who was not wearing red, went to the Australian pavilion and Ashlee, who did not visit the lions, wore the yellow shirt
3. While visiting the King of the Jungle, Chase saw the girl with the red shirt at the monkey exhibit.
4. Paul, who did not like elephants, heard the boy in the purple t-shirt roaring.
5. On her way to visit the monkeys, Michelle passed Steven who was wearing the black t-shirt.

	Elephants	Kangaroos	Lions	Monkeys	Polar Bears	Black	Purple	Red	White	Yellow
Ashlee										
Chase										
Michelle										
Paul										
Steven										
Black										
Purple										
Red										
White										
Yellow										



Magic and Matches

Materials

- sets of 10-20 straws per group and/or match stick worksheet
- sets of small tiles 0-9 (at least 1 set per group better if each student has their own set)

Launch: TW hand out straws and give an arrangement can use #1 on the first worksheet for the students to set up the straws in. Then have the students try to see if by removing 3 straws they can create just 2 squares.

TW hand out the sets of tiles. First ask the students to take out and the 0 tile and only use the 1-9 tiles. Have them try and see if they can find a 3 by 3 square where when you add the top two rows together if they can then get that to equal the 3rd row.

Explore: SW work in small groups 2-4 to come up with a solution. Is there more any one solution? Can then hand out the first worksheet that has easy level problems. (worksheet 2 is intermediate and #3 is the more challenging ones) If students want to continue with this they could try to make there own and then challenge other students to solve them.

SW work individually and in small groups to compete addition problems using the the tiles in a 3 by 3 square. Who can find the largest and smallest sum? Who can find the smallest and largest difference?

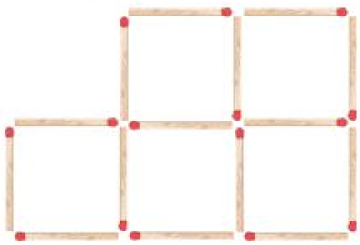
Extension: Next ask the students if they can make a magic square still in a 3 by 3 with all of the rows, columns, and diagonals being the same sum (also could try product). Students can now use number 0-9.

Share: SW share their findings how they got to their answers. If they think they have found all the solutions, the largest or smallest of the solutions. How did you find your answer? What process did you use? Are there any other solutions and how do you know?

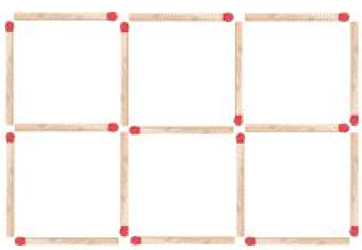
Summarize: TW talk about the different critical thinking techniques that different students were using and point out that there were different ways to solve these problems.

Matchstick Puzzles - #1

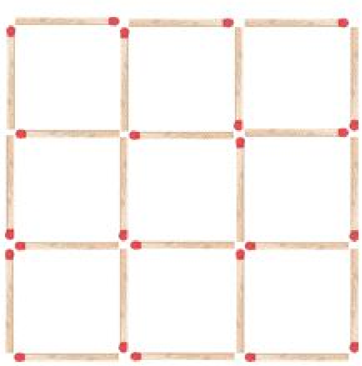
1. Leave just three squares by removing three matchsticks.



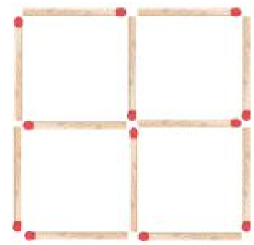
2. Leave just three squares by removing five matchsticks.



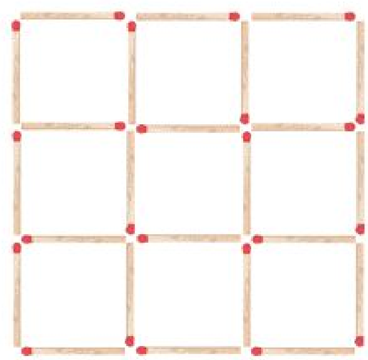
3. Leave just two squares by removing eight matchsticks.



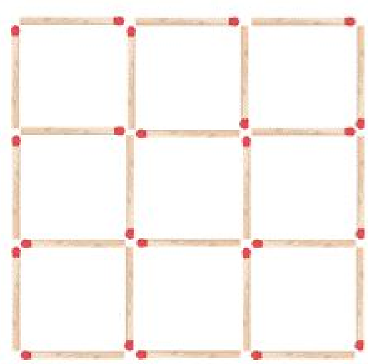
4. Leave just two squares by removing two matchsticks.



5. Leave just six squares by removing eight matchsticks.

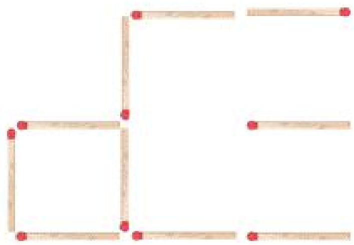


6. Leave just three squares by removing six matchsticks.

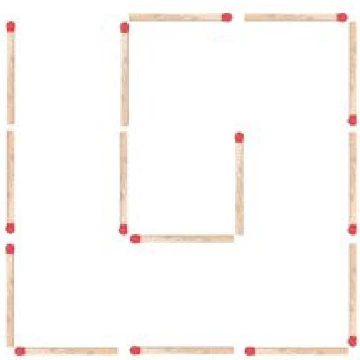


Matchstick Puzzles - #2

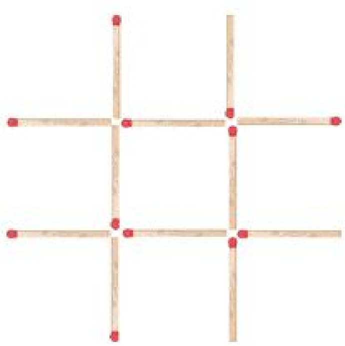
1. Move three matchsticks to make two squares.



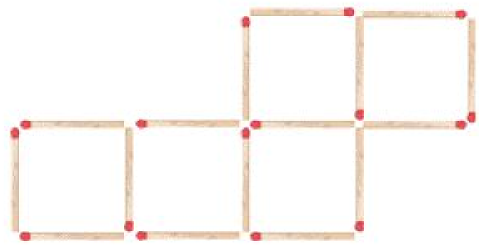
2. Move three matchsticks to make two squares.



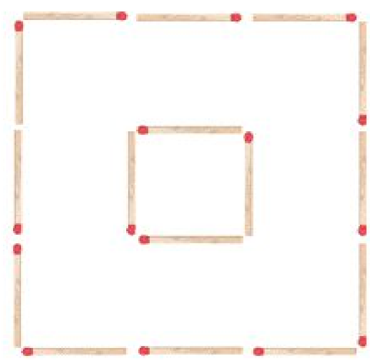
3. Move three matchsticks to make three squares.



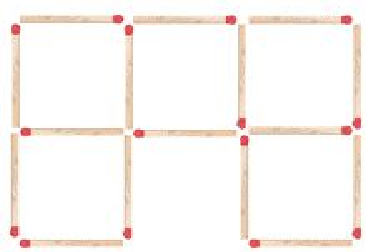
4. Move two matchsticks to make four squares.



5. Move four matchsticks to make three squares.



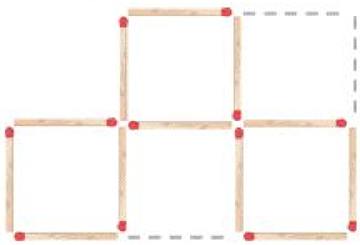
6. Move three matchsticks to make four squares.



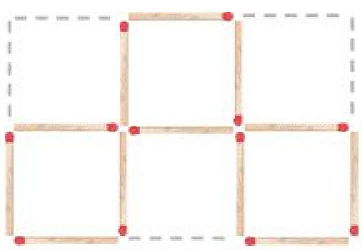
Matchstick Puzzles - #1

SOLUTIONS

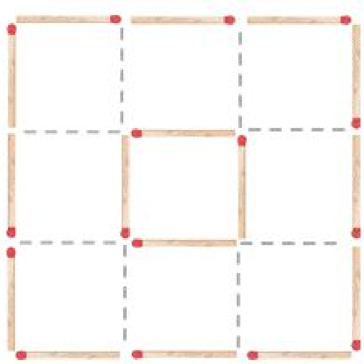
1. Leave just three squares by removing three matchsticks.



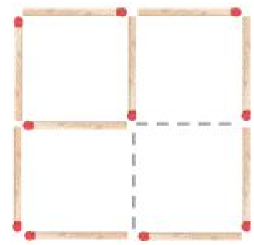
2. Leave just three squares by removing five matchsticks.



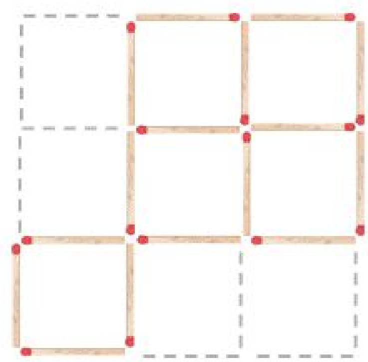
3. Leave just two squares by removing eight matchsticks.



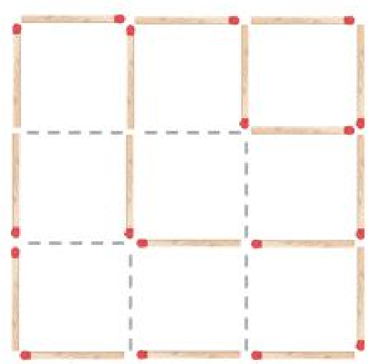
4. Leave just two squares by removing two matchsticks.



5. Leave just six squares by removing eight matchsticks.



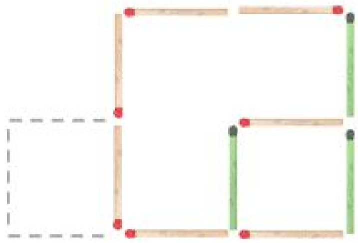
6. Leave just three squares by removing six matchsticks.



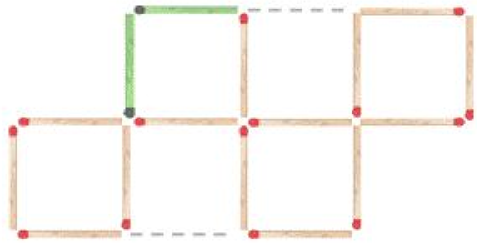
Matchstick Puzzles - #2

SOLUTIONS

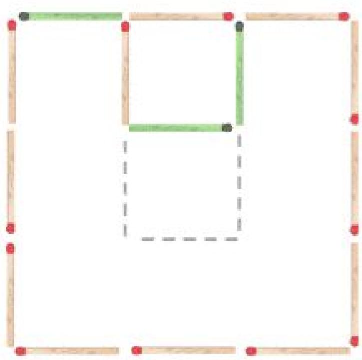
1. Move three matchsticks to make two squares.



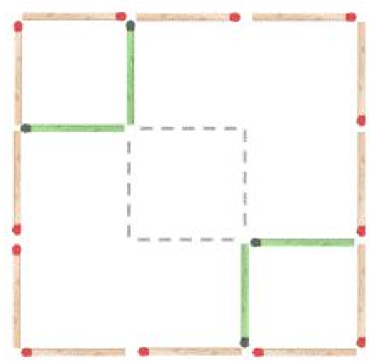
4. Move two matchsticks to make four squares.



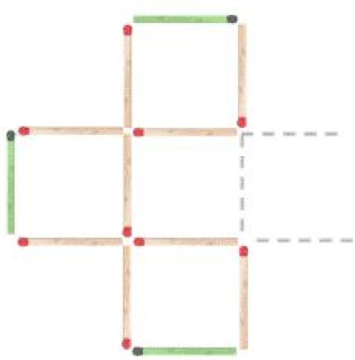
2. Move three matchsticks to make two squares.



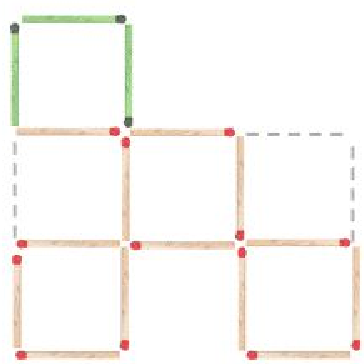
5. Move four matchsticks to make three squares.



3. Move three matchsticks to make three squares.

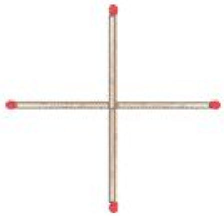


6. Move three matchsticks to make four squares.

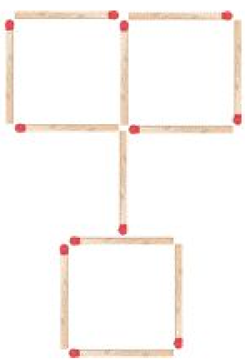


Matchstick Puzzles - #3

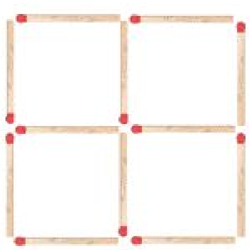
1. Move one matchstick to make a square.



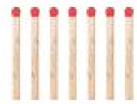
2. Move six matches to make five squares.



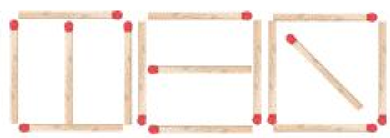
3. Move two matchsticks to make six squares.



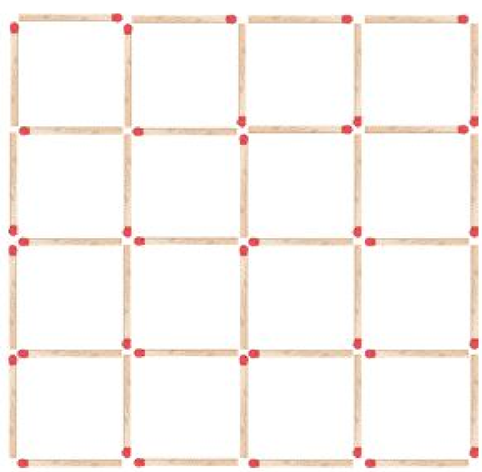
4. Take one away from seven matchsticks, then move two to leave zero.



5. Take away six matchsticks from the fifteen shown to leave ten.



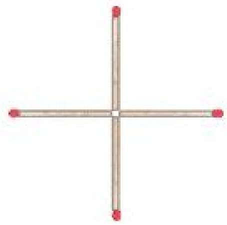
6. Remove 9 matchsticks leaving no square of any size.



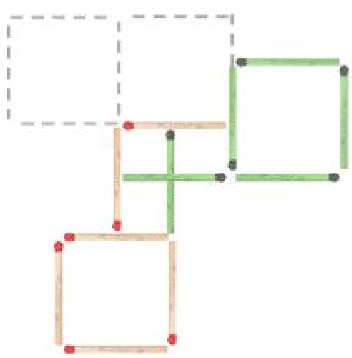
Matchstick Puzzles - #3

SOLUTIONS

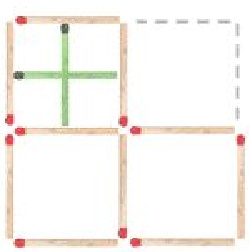
1. Move one matchstick to make a square.



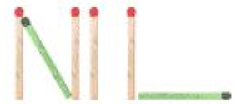
2. Move six matches to make five squares.



3. Move two matchsticks to make six squares.

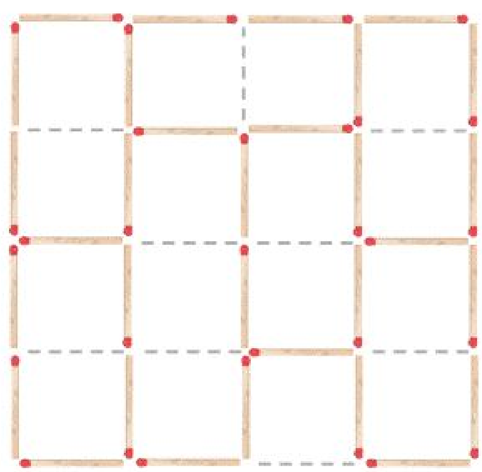


4. Take one away from seven matchsticks, then move two to leave zero.



5. Take away six matchsticks from the fifteen shown to leave ten.

6. Remove 9 matchsticks leaving no square of any size.



Ripple Effect Puzzle

Materials

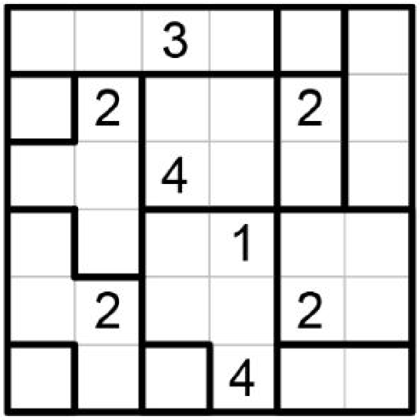
- ripple effect puzzles
- Ivanhoe town map

Launch: TW ask Q: Has anyone every done a Sudoku? The Ripple Effect Puzzle is similar to a Sudoku puzzle. With the difference being that instead of the using the integers 1-9 you are now using the integers 1-n with n being the number of squares in any given region/area. If two or more of the same integer appear in the same row or column then there must be that many spaces or more between those two numbers.

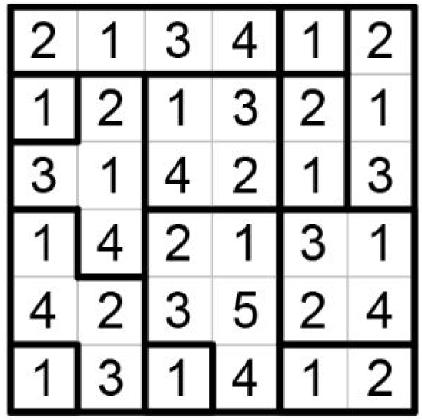
Explore: SW work through different solution paths to try and find the correct unique solution. (each one of the puzzles does have one unique solution)

Share: SW share their process that either did or did not get them to their solutions. Possible outcomes strategies would be
Single squares- What number has to go in a region with a single square?
Eliminate location options- Find a region where a specific number could only be placed in one spot. For example, in the L shaped region to the left and above the number 2 that is placed in this grid, what number can only be placed in one of the three squares?
Eliminate number options- Find a square in a region where only one number could be placed. For example, in the region at the bottom right of the grid, which open square only has one number option remaining and why?

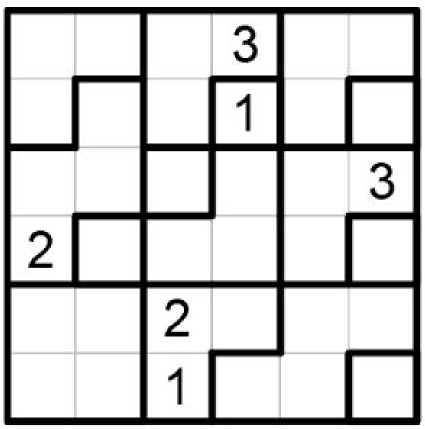
Summarize: TW talk about how the different processes that students came up with and how they all brought the students to one unique solution. The the different solution paths are a good way to look at different ways of thinking about a problem.



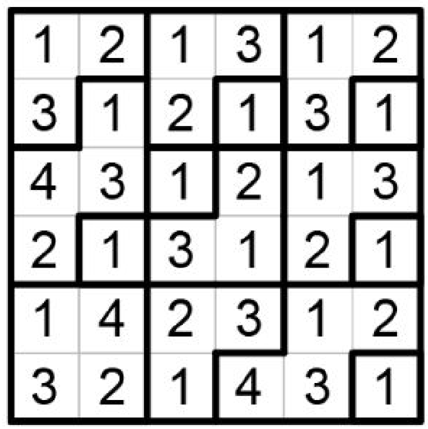
Ripple Effect Example A



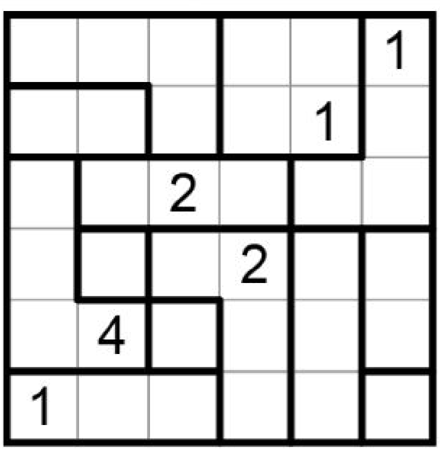
Ripple Effect Example A Solution



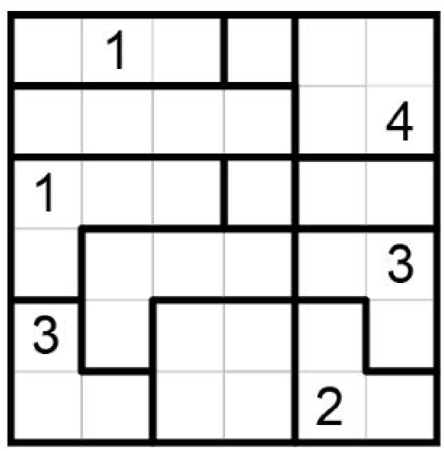
Ripple Effect Example B



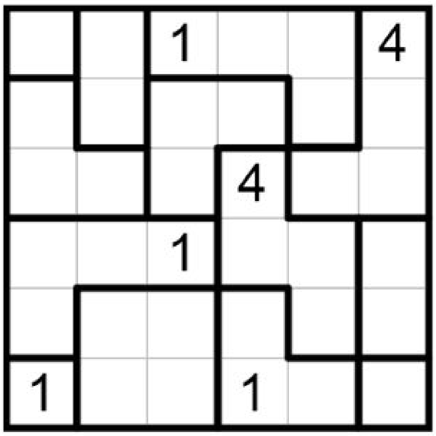
Ripple Effect Example B Solution



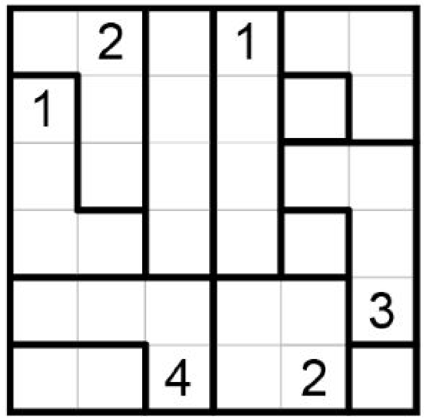
Ripple Effect Puzzle 1



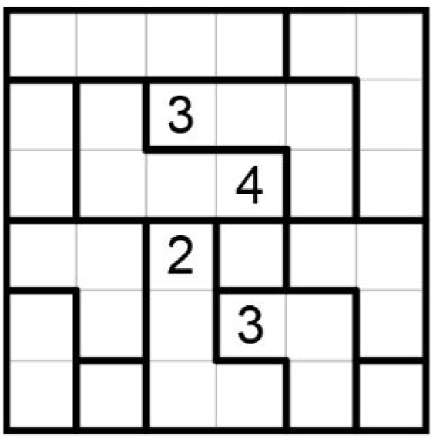
Ripple Effect Puzzle 2



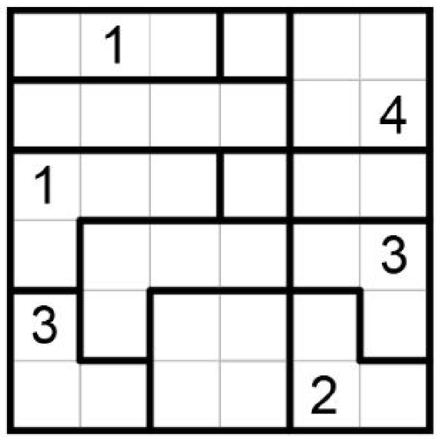
Ripple Effect Puzzle 3



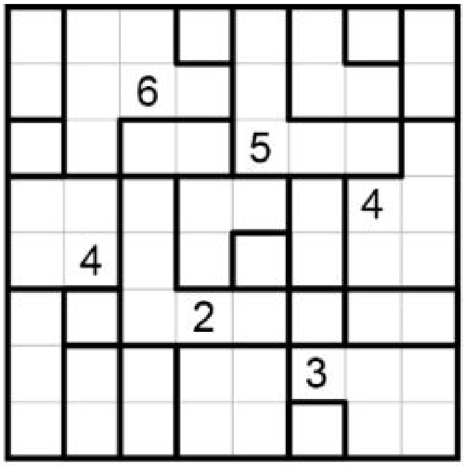
Ripple Effect Puzzle 4



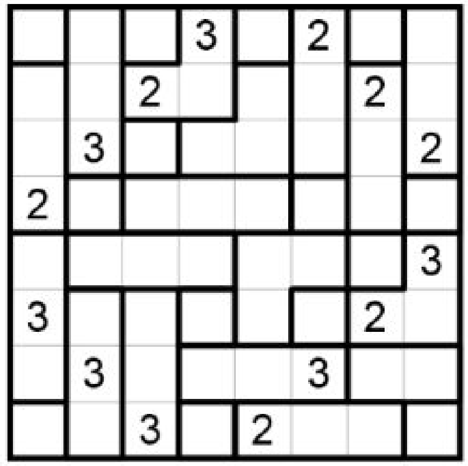
Ripple Effect Puzzle 5



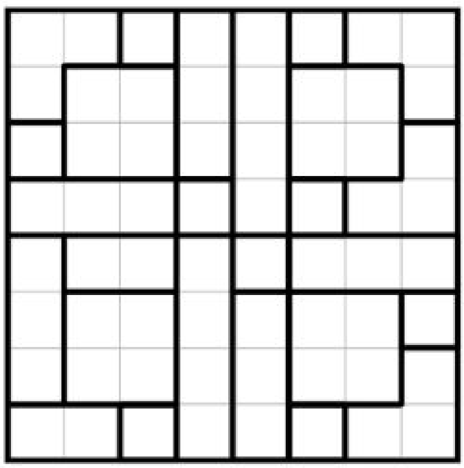
Ripple Effect Puzzle 6



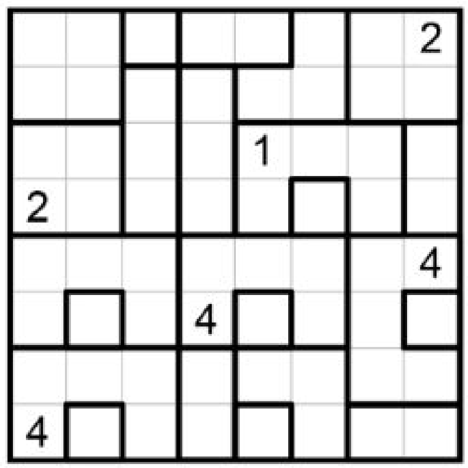
Ripple Effect Puzzle 7



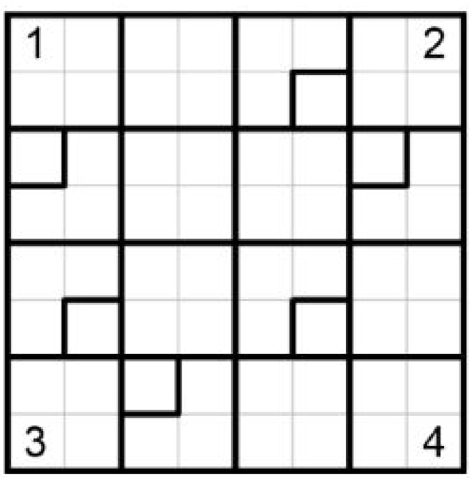
Ripple Effect Puzzle 8



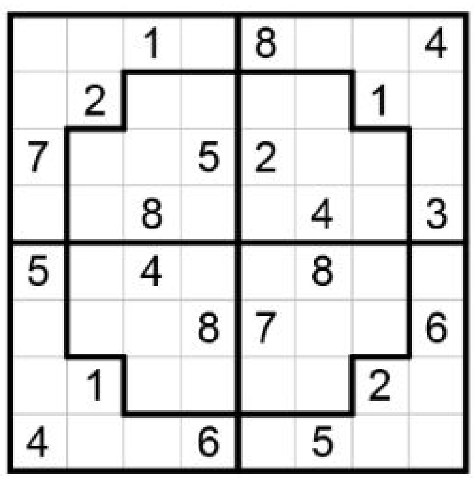
Ripple Effect Puzzle 9



Ripple Effect Puzzle 10



Ripple Effect Puzzle 11



Ripple Effect Puzzle 12

SOLUTIONS

Ripple Effect

2	1	4	3	1	2
3	2	1	4	2	3
1	4	2	1	3	1
2	1	3	2	1	4
4	2	1	3	2	1
3	1	2	1	4	3

Example C

4	2	1	3	2	1
2	1	3	4	1	2
1	3	2	1	4	3
3	1	4	2	3	1
2	4	1	3	1	2
1	2	3	1	2	1

Puzzle 1

3	1	2	1	3	2
4	2	1	3	1	4
1	4	3	1	2	1
2	3	4	2	1	3
3	1	2	4	3	2
1	2	1	3	2	1

Puzzle 2

1	2	1	3	2	4
2	1	3	1	4	2
1	3	2	4	1	3
3	4	1	2	3	1
2	1	4	3	1	2
1	2	3	1	2	1

Puzzle 3

3	2	4	1	3	1
1	4	2	3	1	2
2	1	3	4	2	1
4	3	1	2	1	4
3	1	2	1	4	3
1	2	4	3	2	1

Puzzle 4

4	1	2	3	1	4
1	2	3	1	4	2
2	3	1	4	2	3
3	1	2	1	3	1
1	2	4	3	1	2
2	1	3	1	2	1

Puzzle 5

3	1	2	1	3	2
4	2	1	3	1	4
1	4	3	1	2	1
2	3	4	2	1	3
3	1	2	4	3	2
1	2	1	3	2	1

Puzzle 6

1	2	4	1	2	3	1	2
2	1	6	5	3	1	2	1
1	3	1	2	5	4	1	3
3	2	5	1	2	1	4	2
1	4	1	3	1	2	1	5
2	1	3	2	4	1	2	1
1	2	1	4	2	3	1	2
3	1	2	1	3	1	5	4

Puzzle 7

1	2	1	3	1	2	1	3
3	1	2	1	3	1	2	1
1	3	1	2	1	3	1	2
2	1	3	1	2	1	3	1
1	2	1	3	1	2	1	3
3	1	2	1	3	1	2	1
2	3	1	2	1	3	1	2
1	2	3	1	2	1	3	1

Puzzle 8

1	2	1	3	4	1	2	3
3	4	2	1	3	2	4	1
1	3	1	2	1	3	1	2
2	1	3	1	2	1	3	1
1	2	1	3	1	2	1	3
3	1	2	1	3	4	2	1
2	3	4	2	1	3	1	2
1	2	1	4	2	1	3	1

Puzzle 9

3	4	1	2	1	3	1	2
1	2	3	1	2	1	4	3
4	1	2	3	1	4	2	1
2	3	1	2	5	1	3	2
1	5	4	1	3	2	1	4
3	1	2	4	1	5	2	1
1	2	3	1	2	1	5	3
4	1	5	2	1	3	1	2

Puzzle 10

1	3	2	4	1	3	1	2
2	4	3	1	2	1	3	4
1	2	4	3	1	2	1	3
3	1	2	1	3	4	2	1
1	3	1	2	1	3	4	2
2	1	3	4	2	1	3	1
1	2	1	3	4	2	1	3
3	4	2	1	3	1	2	4

Puzzle 11

4	5	1	3	8	6	7	4
8	2	3	4	6	7	1	5
7	6	1	5	2	3	8	2
6	7	8	2	1	4	5	3
5	2	4	1	3	8	6	7
2	3	5	8	7	2	4	6
3	1	6	7	5	1	2	8
4	8	7	6	1	5	3	4

Puzzle 12

Triangle and Star Sum Puzzles

Materials
- triangle and star puzzle sheets

Launch: TW hand out the the triangle puzzle first and give the students the directions: they may use the digits 1-9 to fill in the circles on the triangle. Each edge of the triangle needs to add up to the same sum. Each number can only be used once.

Explore: SW work in groups and try to find as many solutions that they can using the numbers 1-9. Have you found all of the different solutions? How do you know?
Then you would hand out the Star Puzzle problem and tell them all the lines still need to have the same sum. Now you can use the numbers 1-20. (If students are working on the problem and are having a hard time if one student has come up with the 1-12 star have them share the fact that they only used the number 1-12)

Share: Have student share all of the different solutions that they have found try to fill the board with different solutions. SW share how they found their solutions if they believe there are any other solutions and how they know there are more or that they have found all of them.

Summarize: TW talk about the different solution paths and if we can know that we had them all or not, along with how many solutions there are possible. Also leave this as an on going activity. Where the students can add solutions as the year goes on.

Who is the Winner?

7.4.2.1 Use reasoning with proportions to display and interpret data in circle graphs (pie charts) and histograms. Choose the appropriate data display and know how to create the display using a spreadsheet or other graphing technology.

Material
- Table of an voting outcomes
- Discussion worksheet

Launch: DAY 1 Hand out the voting outcome table and first pose the question of who the students believe should be the winner and their reasons why.
DAY 2 Have students come up to the board and put an x above their 2 favorite numbers 1-50. Then take those numbers and group them 1-5, 6-10, 11-15,...., 41-45, & 46-50. These groups will then become the bars of the histogram.
Ask students to then create a histogram using the 6 different options of ordering that were available for voting for the 3 candidates.

Explore: DAY 1 student work time on coming up with the different ways they have come up with who should win.
DAY 2 Students work in small groups to make the histograms of the different order choices. Does making a set of histograms for each candidate in their 1st, 2nd, and 3rd places as the bars show you anything about who should win.

Share: DAY 1 Students share their winners and how they got to the outcomes they have. Students will have a chance to question others way of coming up with the winner and defend why they believe the winner they came up with is the winner.
DAY 2 Students share how the histograms helped or did not help them confirm their original thoughts on who should be the winner.

Summarize: DAY 1 Give an overview of the different winner and how the students arrived at those winners. TW give names to these ways of finding a winner (Plurality, Elimination, Borda, and Head to Head). If one of these is not a way students came up with a winner TW introduce that way for finding a winner.
DAY 2 Give an overview of how Histograms were used to find the winner and how they can be helpful but also skew the data given in one groups favor.

Order of Choices	Number of Votes
C>M>F	10
C>F>M	30
M>C>F	15
M>F>C	10
F>C>M	5
F>M>C	30

There is going to be a conference held for Math teacher in the Midwest and the conference coordinator wants to find out what city the teachers want the conference to be in. The coordinator gave 3 choices: Chicago, Minneapolis, or Fargo. Each of the 100 teachers surveyed got to rank their choices 1st, 2nd, and 3rd. The table above shows the results of the survey. By the votes where do you think the conference should be held?

Critical Thinking Post-Test Name ________________________

1. Draw a vertex edge graph that has 10 vertices and 7 edges that when you move 2 edges you can make into 2 squares?

2. Show a complete magic square and explain how you got to your answer.

3. Oscar, Peter, Joanne, and Joseph are friends who take the same public bus home.

- When Oscar gets off the bus, two of his friends are still on the bus.
- Joanne gets off the bus before Peter and Joseph.
- Joseph gets off the bus before Peter.

What is the order which the students get off the bus?

4.

Oder of People	Votes
H>P>C	5
H>C>P	11
P>C>H	5
P>H>C	15
C>H>P	10
C>P>H	4

Harry, Peter, and Carrie are all running for class president. When the class of 50 students voted, they rank the 3 candidates their 1st choice, 2nd, and 3rd which is shown in the table above.

a. Who would win using Plurality?

b. Who would win using Elimination?

c. Who would win using Boarda?

d. Who would win using Head to Head?

e. Which of these do you think is the most fair or unfair why?

f. What other way can we show this data to find a winner?